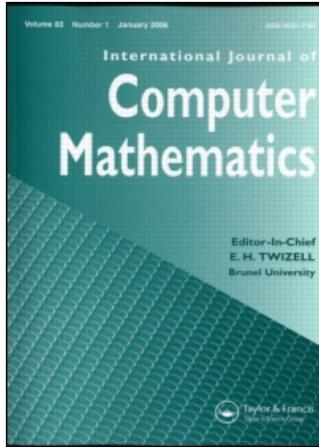


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### A new digital signature scheme based on factoring and discrete logarithms

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## A NEW DIGITAL SIGNATURE SCHEME BASED ON FACTORING AND DISCRETE LOGARITHMS

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This article proposes a new digital signature scheme based on the difficulty of simultaneously factoring a composite number and computing discrete logarithms. In the proposed scheme, each user uses common arithmetic moduli and only owns one private key and one public key. Furthermore, some possible limitations have been analyzed, but none of them can successfully over-rule the proposed scheme.

*Keywords:* Cryptography; Discrete logarithms; Digital signature; Factoring

*C.R. Categories:* E.3.

### 1 INTRODUCTION

Diffie and Hellman [1] were the first to propose the concept of the public-key cryptosystem in 1976. However, several public-key cryptosystems have since been discussed [2–4]. The security of these proposed cryptosystems are based on the following cryptographic assumptions: factoring (FAC) a large composite number [4–7] and solving the discrete logarithm (DL) problem [2, 8–10]. Although these cryptosystems are based on the above cryptographic assumptions that appear secure today, they are very likely to be over-ruled in the future. As soon as some cryptanalyst develops an efficient algorithm to factor the composite number or to compute DL, which could very likely be right around the corner, these cryptosystems will go insecure.

In 1994, Harn [11] first developed a digital signature scheme based on multiple cryptographic assumptions, *i.e.* a mixture of FAC and DL, to enhance the security of the digital signature. Since then many such digital signature schemes have been proposed [12–15]. Unfortunately, all those proposed schemes have proved to be insecure against forgery. In 1996, Lee and Hwang [14] pointed out that Harn's digital signature scheme [11] would not be secure if the DL problem could be solved.

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He and Kiesler's digital signature scheme [12] has also been shown to be insecure [16]. For example, if one has the ability to solve DL [16], then she/he will be able to forge the signature. Also, if one can solve the FAC problem, she/he can obtain the signer's private key.

Recently, Li and Xiao [17] have shown that Shao's digital signature schemes [15] are vulnerable to signature forgery. In addition, Lee [18] has also shown that if the FAC problem can be solved, Shao's digital signature schemes [15] can be broken. Not long ago, He [13, 19] proposed a digital signature scheme based on the difficulty of simultaneously FAC a composite number and computing DLs. He's scheme has three significant advantages.

1. The security of He's scheme is based on the difficulty of simultaneously solving the FAC and the DL problems with arithmetic moduli of almost the same size.
2. Each user has common arithmetic moduli.
3. Each user only owns one private key and one public key.

However, as Sun [20] had pointed out, He's scheme [13] is still not secure against forgery if the DL problem can be solved. To overcome the above limitations a new digital signature is proposed in this paper.

In the next section, we introduce our new digital signature scheme. In Section 3, we analyze the security and performance of the new digital signature scheme and finally, the conclusions are given in Section 4.

## 2 THE PROPOSED DIGITAL SIGNATURE SCHEME

In this section, a new digital signature scheme based on FAC and DL is proposed. The scheme can be divided into three phases: initialization, digital signature generation and digital signature verification.

### 2.1 Initialization

There exists a trusted center whose tasks are to initialize the system and to manage the public directory. First, the trusted center selects the following parameters.

- $P$ : a large prime  $P = 4p_1 \cdot q_1 + 1$ , where  $p_1 = 2p_2 + 1$ ,  $q_1 = 2q_2 + 1$ , and  $p_1, q_1, p_2, q_2$  are all primes.
- $R$ : the product of  $p_1$  and  $q_1$ , *i.e.*  $R = (P - 1)/4 = p_1 \cdot q_1$ .
- $g$ : a primitive element with order  $p_1 \cdot q_1$ .
- $h(\cdot)$ : a one-way hash function, *e.g.* MD5 or SHA-1.

The parameters  $P$  and  $g$  are used by all users, and  $p_1, q_1, p_2$  and  $q_2$  are all discarded. Each user owns a private key  $x \in Z_R$  such that  $\gcd(x^2, R) = 1$  and the corresponding public key  $y$  which is certified by the certificate authority as

$$y = g^{x^2+x^{-2}} \bmod P. \quad (1)$$

### 2.2 Digital Signature Generation

To sign a message  $M$ , the signee carries out the following steps.

1. Randomly select an integer  $t_1$  and compute  $k$  as

$$k = t_1 x \bmod R. \quad (2)$$

2. Compute

$$t_2 = kx \bmod R, \quad (3)$$

$$r = g^{t_1+t_2} \bmod P. \quad (4)$$

3. Calculate  $s_1$  and  $s_2$  under the conditions

$$s_1 = h(r, M)t_1 + x \bmod R, \quad (5)$$

$$s_2 = h(r, M)t_2 - x^{-1} \bmod R. \quad (6)$$

Then  $(r, s_1, s_2)$  is a signature of  $M$ . The signee then sends  $(r, s_1, s_2)$  to the verifier.

### 2.3 Digital Signature Verification

On receiving the digital signature  $(r, s_1, s_2)$  the verifier can confirm the validity of the digital signature by the following equation

$$g^{s_1^2+s_2^2} = r^{h^2(r,M)} y \bmod P. \quad (7)$$

If it holds, then  $(r, s_1, s_2)$  is a valid signature of  $M$ .

**THEOREM 2.1** *If the signee follows the above digital signature protocol, the verifier always accepts the digital signature.*

*Proof* Squaring Eqs (5) and (6)

$$s_1^2 = h^2(r, M)t_1^2 + 2h(r, M)t_1x + x^2 \bmod R,$$

$$s_2^2 = h^2(r, M)t_2^2 - 2h(r, M)t_2x^{-1} + x^{-2} \bmod R.$$

Summing the above 2 equations

$$s_1^2 + s_2^2 = h^2(r, M)(t_1^2 + t_2^2) + 2h(r, M)(t_1x - t_2x^{-1}) + (x^2 + x^{-2}) \bmod R.$$

According to Eqs (2) and (3)

$$s_1^2 + s_2^2 = h^2(r, M)(t_1^2 + t_2^2) + (x^2 + x^{-2}) \bmod R.$$

Therefore we have

$$g^{s_1^2+s_2^2} = g^{h^2(r,M)(t_1^2+t_2^2)+(x^2+x^{-2})} \bmod P$$

or

$$g^{s_1^2+s_2^2} = r^{h^2(r,M)} y \bmod P.$$

The above equation is equivalent to Eq. (7). With the knowledge of the signee's public key  $y$  and the signature  $(r, s_1, s_2)$  of  $M$ , the verifier can authenticate the message  $M$  because the verifier can be convinced that the message was really signed by the signee. Else, the signature  $(r, s_1, s_2)$  is invalid.

### 3 SECURITY AND PERFORMANCE ANALYSIS

The security of the proposed scheme is based on the cryptographic assumptions of intractability of FAC and DLs. In this section, we analyze some possible limitations of our digital signature scheme.

**Limitation 1:** An adversary attempts to derive the private key from the corresponding public key for any user.

**Analysis of limitation 1:** For the adversary, to recover a private key  $x$  from Eq. (1) is polynomially equivalent to figuring out both FAC and DL simultaneously. She/he also has to solve the DL problem to obtain  $(x^2 + x^{-2}) \bmod R$ ; at the same time, she/he also has to solve the FAC problem to obtain a private key  $x$  from  $(x^2 + x^{-2}) \bmod R$ .

**Limitation 2:** An adversary attempts to derive the private key from a valid signature  $(r, s_1, s_2)$  of message  $M$ .

**Analysis of limitation 2:** To obtain a private key  $x$  or to compute a private key  $x$  for  $x^{-1}$  from Eqs (5) and (6), the adversary has to know  $t_1$  or  $t_2$ . Given  $P, g, r, s_1$  or  $P, g, r, s_2$  to obtain  $t_1$  or  $t_2$  from Eq. (4) is non-feasible under FAC and DL assumptions.

**Limitation 3:** An adversary attempts to forge a valid signature  $(r, s_1, s_2)$  for message  $M$ . In this case, the adversary has to find a three-tuple  $(r, s_1, s_2)$  to satisfy Eq. (7).

**Analysis of limitation 3:**

1. The adversary randomly chooses  $r$  and  $s_1$ . She/he then has to solve the problem of finding  $s_2$  from  $\alpha$  where

$$\begin{aligned}\alpha &\equiv r^{h(r,M)^2} y g^{-s_1^2} \bmod P, \\ &\equiv g^{s_2^2} \bmod P.\end{aligned}$$

This is equivalent to solving both the FAC and the DL at the same time.

2. The adversary randomly chooses  $r$  and  $s_2$ . She/he has to solve the problem of obtaining  $s_1$  from  $\beta$  where

$$\begin{aligned}\beta &\equiv r^{h(r,M)^2} y g^{-s_2^2} \bmod P, \\ &\equiv g^{s_1^2} \bmod P.\end{aligned}$$

This is as difficult as solving FAC and DL simultaneously.

3. The adversary randomly chooses  $s_1$  and  $s_2$ . She/he has to solve the problem of deriving  $r$  from  $\gamma$  where

$$\begin{aligned}\gamma &\equiv g^{s_1^2 + s_2^2} y^{-1} \bmod P, \\ &\equiv r^{h(r,M)^2} \bmod P.\end{aligned}$$

This is as hard as distinguishing between the FAC, DL and one-way hash function assumptions.

4. The adversary randomly chooses a three-tuple  $(r, s_1, s_2)$ . This is also as difficult as solving the FAC, DL and one-way hash functions simultaneously.

Therefore, we conclude that the task of forging the message is more difficult than solving the problems of FAC and DLs simultaneously.

Next, we analyze the computational cost of the proposed scheme. In the performance evaluation  $T_{\text{exp}}$  represents the time for a modular exponentiation computation,  $T_{\text{inv}}$  is the time for a modular inverse computation, and  $T_h$  is the time for computing a one-way hash function  $h(\cdot)$ .

We ignore the time for performing modular multiplication and addition. Then, the total computational complexities of the signer and the verifier are  $T_{\text{exp}} + T_{\text{inv}} + T_h$  and  $2T_{\text{exp}} + T_h$ , respectively.

## 4 CONCLUSION

A new digital signature scheme has been proposed in this paper. The security of the proposed scheme is equivalent to solving both the FAC problem and the DL problem. Some possible limitations have also been considered. The advantages of the proposed scheme are the following three characteristics: (i) the sizes of the arithmetic moduli for FAC and DLs are almost the same; (ii) each user uses common arithmetic moduli; and (iii) each user only owns one private key and one public key.

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