

Cryptanalysis of the Batch Verifying Multiple DSA-type Digital Signatures *

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Abstract

To reduce the signature verification time, Harn proposed an efficient batch verifying multiple DSA-type digital signatures instead of verifying each individual digital signature separately. However, there is a weakness in his scheme. In this article, we present our attack to against his scheme.

Keywords: Cryptography, digital signature, DSA.

1 Introduction

In digital signature, a sender signed a document using his/her private key and a receiver can verify this signature by the sender's public key. When the sender signed t documents for the receiver, the sender need to generate t signatures and the receiver need to verify these signatures in t times. This is inefficient to verify those signatures for the receiver. In 1994, Naccache et al. [7] proposed an efficient scheme to batch verify multiple DSA digital signatures. The multiple DSA digital signatures are batch verified by the receiver which requires only one times. However, this scheme is insecure [6]. In 1995, Harn proposed a DSA-type secure interactive batch verification protocol [1]. In 1998, Harn proposed two types of efficient non-interactive batch verification protocol: DSA-type and RSA-type multiple digital signatures [2, 3]. However, the batch verifying multiple RSA-type digital signatures [3] is insecure [5].

The batch verifying multiple DSA-type digital signatures [2] can against Lim and Lee's attack [6]. This scheme is based on the DSA-type digital signature [4]. The main advantage of this scheme is that it reduces signature verification time. It is more efficient than verify each individual signature

separately. However, there is a weakness in his scheme. In this article, we will show that an attacker can easily forge individual signatures and make the batch verification valid.

2 Rieview

In this section, we review the Harn's DSA-type digital signature [4] and batch verifying multiple DSA-type digital signature [2].

DSA-type Digital Signature:

The parameters of DSA-type digital signature are composed by public information p, q, g , a public key y and a secret key x , where p is a large prime, q is a factor of $(p - 1)$, g is generated from $\text{GF}(p)$, x is a random number less than q , and y is computed by $y = g^x \text{ mod } p$.

Assume that Alice wants to send message M and its digital signature (r, s) to Bob. The sender, Alice, need to generate the digital signature (r, s) as follows. First, Alice generates a random number, k , less than q . Next, Alice computes r and s as follows.

$$r = (g^k \text{ mod } p) \text{ mod } q, \quad (1)$$

$$s = rk - Mx \text{ mod } q. \quad (2)$$

Whenever Bob receives (M, r, s) from Alice, he verifies the correctness of the signature on the message M by checking whether the equation $r = (g^{sr^{-1}} y^{Mr^{-1}} \text{ mod } p) \text{ mod } q$ holds.

Multiple DSA-type Digital Signature:

Assume that Alice wants to send M_1, M_2, \dots, M_t and its digital signatures $(r_1, s_1), (r_2, s_2), \dots, (r_t, s_t)$ to Bob. All digital signatures satisfy the following

equation:

$$r_i = (g^{s_i r_i^{-1}} y^{M_i r_i^{-1}} \bmod p) \bmod q, \quad i = 1, 2, \dots, t. \quad (3)$$

Whenever Bob receives these multiple signatures from Alice, Bob verifies the correctness of these multiple signatures on messages M_1, M_2, \dots, M_t by checking the following equation:

$$\prod_{i=1}^t r_i = (g^{\sum_{i=1}^t s_i r_i^{-1}} y^{\sum_{i=1}^t M_i r_i^{-1}} \bmod p) \bmod q. \quad (4)$$

By verifying the above equation, Bob can verify the multiple signatures of messages M_1, M_2, \dots, M_t are signed by Alice. Harn's scheme is efficient to batch verifying multiple DSA-type digital signatures. However, there is a weakness in his scheme. We show that the weakness in next section.

3 The Weakness of Harn's Scheme

In this section, we show that the signer, Alice, can forge individual signatures and make a false batch verification valid.

We assume that the signer, Alice, sends messages and its forged signatures to the receiver Bob. Let $s'_i = s_i + a_i r_i \bmod q, i = 1, 2, \dots, t$, where a_i is an integer such that $\sum_{i=1}^t a_i = 0$. Alice sends the forged pairs (M_i, r_i, s'_i) , $i = 1, 2, \dots, t$, to Bob. Since these multiple signatures satisfy the verification in Equation 4, Bob is convinced that these message are signed by the dishonest user, Alice.

For example, Alice forges three signatures $(r_1, s'_1), (r_2, s'_2), (r_3, s'_3)$ of messages M_1, M_2, M_3 , respectively. Here, $s'_1 = s_1 + 2r_1 \bmod q, s'_2 = s_2 + 3r_2 \bmod q, s'_3 = s_3 - 5r_3 \bmod q$, and $s_i = r_i k_i - M_i x \bmod q, r_i = (g^{k_i} \bmod p) \bmod q, i = 1, 2, 3$. Then Alice sends $(M_1, r_1, s'_1), (M_2, r_2, s'_2)$, and (M_3, r_3, s'_3) to Bob. Bob can verify the correctness of the signature on the message M_1, M_2 , and M_3 by checking the following equation:

$$\begin{aligned}
r_1 r_2 r_3 &= (g^{s'_1 r_1^{-1} + s'_2 r_2^{-1} + s'_3 r_3^{-1}} y^{M_1 r_1^{-1} + M_2 r_2^{-1} + M_3 r_3^{-1}} \bmod p) \bmod q. \\
&= (g^{(s_1 + 2r_1)r_1^{-1} + (s_2 + 3r_2)r_2^{-1} + (s_3 - 5r_3)r_3^{-1}} y^{M_1 r_1^{-1} + M_2 r_2^{-1} + M_3 r_3^{-1}} \bmod p) \bmod q. \\
&= (g^{s_1 r_1^{-1} + s_2 r_2^{-1} + s_3 r_3^{-1}} y^{M_1 r_1^{-1} + M_2 r_2^{-1} + M_3 r_3^{-1}} \bmod p) \bmod q. \tag{5}
\end{aligned}$$

Since the above equation holds, Bob believes that the signatures (r_1, s'_1) , (r_2, s'_2) , and (r_3, s'_3) are valid signatures of messages M_1 , M_2 , and M_3 , respectively. However, Alice can deny she had signed these messages to Bob, because $r_i \neq (g^{s'_i r_i^{-1}} y^{M_i r_i^{-1}} \bmod p) \bmod q$, $i = 1, 2, 3$.

Although the DSA-type digital signature is a secure scheme to produce individual signature [4], the batch verifying multiple DSA-type digital signatures [2] is insecure.

4 Conclusion

Harn proposed an efficient batch verifying multiple DSA-type digital signatures instead of verifying each individual digital signature separately. However, we presented a weakness of his scheme in this letter. We have shown that a signer can easily forge his/her signatures and the receiver cannot discover that the signatures are illegal in Harn's scheme.

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